

Anisotropy of the free path length is introduced as a concept on the basis of which the two-dimensional steady flow of the simple wave kind (Prandtl-Meyer flow) can be analyzed for any power-law interaction between molecules.

In the study of rapidly expanding gas streams (discharge of a jet into vacuum [1, 2], spherically symmetrical evaporation of a comet head [3], etc.), when the medium passes through all stages from a continuous one to a collisionless one, it is generally necessary to solve equations of kinetics [4-6] and this is rather difficult, especially for an arbitrary flow geometry. A simple method of analysis has been proposed in [7] in which the concept of the mean free path length  $l_1$  is generalized and which consists of constructing an  $l$ -surface ( $l$  denoting the path length anisotropy) whose variance from the  $l_1$ -sphere would represent the degree of local anisotropy. This generalization seems unjustified in the sense that anisotropy occurs noticeably within the flow regions where the gas-dynamic parameters vary appreciably along the path length. Thus, this method is, on the one hand, similar to the method of the indirect proof: the presence of a continuous medium at a given point is first assumed and then denied by the very shape of the  $l$ -surfaces, while, on the other hand, this method may be treated as the zeroth approximation in some iteration process of solving the relaxation model of the Boltzmann equation [8, 9] with the mean time between collisions  $\tau$  or the mean (already anisotropic now) path length  $l = \tau \langle c \rangle$  on the right-hand side. The effectiveness of this method has been established by comparing the "boundary" of a continuous medium with the "frozen in" values of the temperature and the Mach number obtained: a) by the method of  $l$ -surfaces [2, 7]; b) by the solution of the Boltzmann equation for a spherical source [5]; and c) from experimental results [10].

In this article we will use the path length anisotropy for analyzing the two-dimensional classical Prandtl-Meyer flow, where the gas-dynamic parameters remain constant along the  $\varphi = \text{const}$  line in the polar system of coordinates [11].

Let a simple inviscid and thermally nonconducting gas flow along a semiinfinite wall  $-\infty < r < 0$ ,  $\varphi = 3/2\pi$  (Fig. 1) at a constant velocity  $V_n \geq a_n$  ( $Ma_n \geq 1$ ). Beyond the edge of the wall it expands into vacuum and turns by some angle  $\varphi^1 < 3/2\pi$  so that a gasless region (from the standpoint of the model of a continuous medium) is maintained between the wall and the last flow line.

Let a sample molecule move from point  $r$ ,  $\varphi$  in the chosen system of coordinates, at a velocity  $\bar{\xi} = \bar{V} + \bar{c}$  in the  $x$ -direction in this system or in the  $y$ -direction in the satellite system of coordinates moving relative to the first one at a local macroscopic velocity  $\bar{V}(r, \varphi)$ . Then  $n_1 \langle \sigma g \rangle_1$  is the frequency of collisions between the sample molecule and field molecules at point 1 ( $\langle \rangle_1$  is the operator of space averaging the thermal velocities of field molecules). The expression in [7]

$$\int_0^l n_1 \langle \sigma g \rangle_1 \frac{dy}{c} = 1 \quad (1)$$

is the equation of the anisotropic path length for a sample molecule moving at the velocity  $\bar{c}$  in the satellite system of coordinates, and it becomes the well-known relation  $n\sigma_0\sqrt{2}l_1 = 1$  for an isotropic path length  $l_1$  (if  $c = \langle c \rangle$  is chosen) in the case of a homogeneous quiescent gas of rigid molecules ( $\sigma = \sigma_0$ ).

If the molecules have a concentric force field  $F_{ij} \sim r_{ij}^{-s}$ , then  $\sigma \sim g^{-4/(s-1)}$  so that  $\sigma g \sim g^\beta$ , where  $\beta = (s-5)/(s-1)$ . We will also assume, for simplicity, the field molecules to have a thermal velocity

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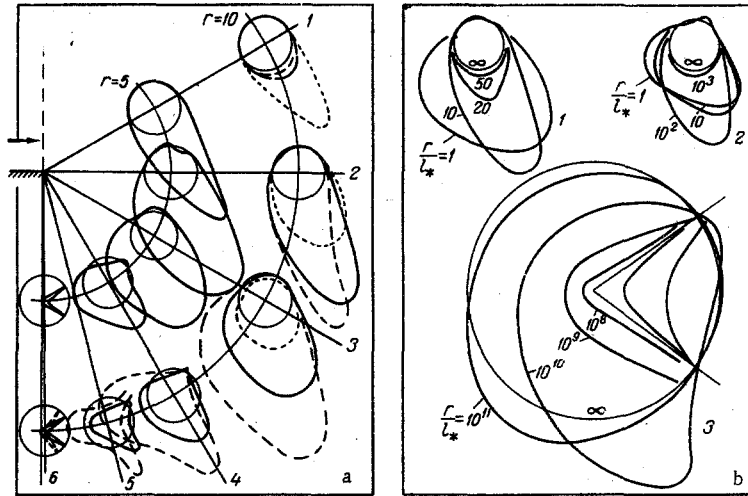


Fig. 3. Lines  $l/l_i$  for rigid molecules. Near the edge of the half-plane (a): 1)  $\varphi = 1/3\pi$  and  $l_i/l_* = 1.54$ ; 2)  $1/2\pi$  and 2.82; 3)  $2/3\pi$  and 8.0; 4)  $5/6\pi$  and 57.68; 11/12 $\pi$  and 449.7; 6)  $(1 - 10^{-2})\pi$  and  $2.06 \cdot 10^6$ . Along the  $\varphi = \text{const}$  rays: 1)  $\varphi = 1/2\pi$  and  $l_i/l_* = 2.82$ ; 2)  $3/4\pi$  and 17.84; 3)  $(1 - 10^{-2})\pi$  and  $2.06 \cdot 10^6$  (magnified for clarity). Dotted line corresponds to  $q = 0.575$ , solid line corresponds to  $q = 1.0$ , and dashed line corresponds to  $q = 1.425$ .

A computer-aided solution of Eq. (2) for  $l/r$  has been carried out for flow regions far from the boundary with the vacuum (where  $\alpha$  is not small). Typical results for monoatomic Maxwellian and rigid molecules are shown in Figs. 2 and 3 in the form of maps of lines representing the relative path length  $l/l_i$  of sample molecules whose mean velocity is  $\langle c \rangle$ . For one of the radii  $r/l_* = 10$ , these path lengths have been plotted (dashed and dotted curves) also for molecules with velocities corresponding to the dispersion boundaries  $c = \langle c \rangle \pm D = q\langle c \rangle$ ,  $D^2 = \langle c^2 \rangle - \langle c \rangle^2 = \langle c \rangle^2(3\pi/8 - 1)$  ( $q = 1.425$  and  $0.575$ ).

It is evident that for Maxwellian molecules the  $l$ -surfaces stretch considerably toward the vacuum while the exit point approaches the classical boundary  $\varphi^1 = \pi$  ( $\alpha \rightarrow 0$ ) $\pi$  which has to do with a decrease in the gas density along this direction. (We must bear in mind that the  $l$ -surfaces in Figs. 2 and 3 have been plotted in the satellite system of coordinates and, indeed, lie inside the continuous medium to the left of the  $\varphi = \varphi^1 = \pi$  ray.)

In the case of rigid molecules, the shape of the  $l$ -surfaces depends also on the relative velocity of the sample molecule; this fact, together the decrease in the density along  $\varphi$ , makes the  $l(\theta)/l_i$  curves more unpredictable. When  $\varphi = \text{const}$ , they more and more resemble a circle as the radius increases; when  $r = \text{const}$  and  $\varphi \rightarrow \varphi^1 = \pi$ , they shrink on the average (although  $l \sim \alpha^{-2}$  increases) and they become sharply peaked and stretched under the angle  $\theta = 90 \pm 36^\circ$  toward the continuous medium. When  $c = \langle c \rangle + D$  and  $q = 1.425$  ("hot" boundary of the dispersion region), there appears a pronounced elongation toward the vacuum ( $\theta \approx 3/2\pi$ ), which indicates a tendency of sample molecules ( $q > 1$ ) to leave the continuous medium.

#### NOTATION

$\sigma, m$	are the collision section and mass of a molecule, respectively;
$s$	is the exponent in the power-law relation between interaction force and intermolecular distance;
$\bar{c}, \bar{\xi}, g =  \bar{\xi}_1 - \bar{\xi} $	are the thermal, total, and relative velocity of a molecule;
$n, \rho, T, V$	are the numerical and mass per volume density, temperature, and macroscopic velocity of the gas, respectively;
$a$	is the velocity of sound;
$\kappa$	is the ratio of specific heats;
$r, \varphi, z$	are the cylindrical coordinates tied to the flow around a half-plane ( $z$ -axis along the edge);
$\beta = (s - 5)/(s - 1)$ ;	
$q = c/\langle c \rangle$ ;	
$Ma = V/a$	is the Mach number;

$$\begin{aligned} \sigma_i &= \varphi_i + \alpha_i - \pi/2; \\ \alpha_i &= \arcsin M_i^{-1}; \\ \varphi^0 &= \pi/2 - \alpha_i; \\ \varphi_i &= \left(\frac{\kappa+1}{\kappa-1}\right)^{1/2} \arcsin \left[ \frac{1/2(\kappa-1)(M_n^2-1)}{1+1/2(\kappa-1)M_n^2} \right]^{1/2}; \\ \varphi^1 &= \varphi_m - \sigma_i; \\ \varphi_m &= \frac{\pi}{2} \left(\frac{\kappa+1}{\kappa-1}\right)^{1/2}; \\ \langle c \rangle &= \left(\frac{8kT}{\pi m}\right)^{1/2}. \end{aligned}$$

### Subscripts

- \* refers to the stagnation point;
- i refers to the entering stream.

### Superscript

- 1 refers to a point along the path of a sample molecule.

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